Sesión Especial 19

Matemática Discreta

Organizadores

- Delia Garijo (Universidad de Sevilla)
- Vera Sacristán (Universitat Politècnica de Catalunya)

Descripción

El objetivo de la sesión es presentar una visión amplia de las áreas más activas de investigación en matemática discreta en España, cubriendo sin excluir otros: combinatoria, métodos probabilísticos, teoría de grafos, geometría discreta, geometría computacional y aplicaciones. La sesión ha sido diseñada para combinar la experiencia de investigadores de sólida trayectoria y reconocido prestigio internacional con la potenciación de jóvenes investigadores del área.

Programa

Lunes, 4 de febrero (mañana)

11:30 – 12:30	Marc Noy (Universitat Politècnica de Catalunya, Barcelona Graduate School of Mathematics) *Lógica, combinatoria y probabilidad*
12:30 – 13:00	Aida Abiad (Maastricht University and Ghent University) *Proof of a conjecture of Graham and Lovász concerning unimodality of coefficients of the distance characteristic polynomial of a tree*
13:00 – 13:30	Shalom Eliahou (Université du Littoral Côte d'Opale) *Generic numerical semigroups*

Lunes, 4 de febrero (tarde)

17:00 – 18:00	Rodrigo I. Silveira (Universitat Politècnica de Catalunya) *Análisis de trayectorias: algoritmos y desafíos*
18:00 – 18:30	Miriam Pisonero (Universidad de Valladolid) *Why* $\{6, 3, 3, -5, -5\}$ *is the spectrum of a weighted digraph but not of a weighted graph?*
18:30 – 19:00	David Orden (Universidad de Alcalá) *Discrete mathematics apply as time goes by*

Martes, 5 de febrero (mañana)

11:30 – 12:30	Alberto Márquez (Universidad de Sevilla) *Los escutoides: mi experiencia en un trabajo multidisciplinar*
12:30 – 13:00	Anna de Mier (Universitat Politècnica de Catalunya) *On Stanley's tree isomorphism problem and the rooted U-polynomial*
13:00 – 13:30	Irene Parada (Graz University of Technology) *A superlinear lower bound on the number of 5-holes*

Lógica, combinatoria y probabilidad

Marc Noy

Universitat Politècnica de Catalunya
Barcelona Graduate School of Mathematics

marc.noy@upc.edu

Resumen. Presentamos resultados recientes [4, 1, 5] de leyes 0–1 y leyes de convergencia en lógica de primer orden y lógica monádica de segundo orden para varias clases de grafos aleatorios. Las demostraciones utilizan resultados analíticos y probabilísticos [2, 3], además de resultados de combinatoria y lógica. Para seguir la exposición no es necesario un conocimiento previo de lógica.

Referencias

[1] A. Atserias, S. Kreutzer, M. Noy. On Zero-One and Convergence Laws for Graphs Embeddable on a Fixed Surface. In Proc. of ICALP, *International Colloquium on Automata, Languages and Programming*, Prague, 2018. Available at `https://iuuk.mff.cuni.cz/ icalp2018/`

[2] G. Chapuy, E. Fusy, O. Giménez, B. Mohar, M. Noy. Asymptotic enumeration and limit laws for graphs of fixed genus. *J. Combin. Theory Ser. A* 118 (2011), 748–777.

[3] O. Giménez, M. Noy, J. Rué. Graph classes with given 3-connected components: asymptotic enumeration and random graphs. *Random Structures Algorithms* 42 (2013), 438–479.

[4] P. Heinig, T. Müller, M. Noy, A. Taraz. Logical limit laws for minor-closed classes of graphs. *J. Combin. Theory Ser. B* 130 (2018), 158–206.

[5] M. Noy, T. Müller. The first order convergence law fails for random perfect graphs. *Random Structures Algorithms* (to appear).

Proof of a conjecture of Graham and Lovász concerning unimodality of coefficients of the distance characteristic polynomial of a tree

Aida Abiad

Maastricht University and Ghent University

a.abiadmonge@maastrichtuniversity.nl

Abstract. The 40-years-old conjecture of Graham and Lovász that the (normalized) coefficients of the distance characteristic polynomial of a tree are unimodal is proved; it is also shown that the (normalized) coefficients are log-concave. Upper and lower bounds on the location of the peak are established. New research directions will be discussed.

Joint work with G. Aalipour, Z. Berikkyzy, L. Hogben, F.H.J. Kenter, J.C.-H. Lin and M. Tait
Research supported by Elsevier Mathematical Sciences Sponsorship Fund and NSF Grant NSF 1500662

Generic numerical semigroups

Shalom Eliahou

Université du Littoral Côte d'Opale

eliahou@univ-littoral.fr

Abstract. A *numerical semigroup* S is a cofinite submonoid of $\mathbb{N}$, i.e. a subset $S \subseteq \mathbb{N}$ containing 0, stable under addition and with finite complement in $\mathbb{N}$. The *genus* of S is $g = |\mathbb{N} \setminus S|$, the *multiplicity* of S is $m = \min S \setminus \{0\}$, and the *conductor* of S is the least integer c such that S contains $c + \mathbb{N}$. We say that S is *generic* if $c \leq 3m$. This terminology is motivated by a 2013 result of Zhai, according to which *the proportion of generic numerical semigroups of genus g among all numerical semigroups of that genus tends to* 1 *as g goes to infinity.* In this talk, we shall review some open problems on numerical semigroups together with some partial solutions in the generic case.

Referencias

[1] S. Eliahou, Wilf's conjecture and Macaulay's theorem, J. Eur. Math. Soc. 20 (2018) 2105–2129. DOI 10.4171/JEMS/807.

Joint work with Jean Fromentin

Análisis de trayectorias: algoritmos y desafíos

Rodrigo I. Silveira

Universitat Politècnica de Catalunya

rodrigo.silveira@upc.edu

Resumen. La creciente popularidad de dispositivos móviles equipados con tecnología de geolocalización, como por ejemplo receptores GPS (*Global Positioning System*), ha hecho que la generación de datos sobre movimientos sea más fácil que nunca. Una *trayectoria* describe la variación de la ubicación geográfica de un objeto a lo largo del tiempo, típicamente, de manera discreta, a través de una serie de puntos en el espacio, obtenidos cada cierto tiempo. Las trayectorias son objetos geométricos que requieren de algoritmos geométricos para ser procesadas y analizadas. En esta charla repasaremos algunos de los problemas algorítmicos relacionados con el análisis de trayectorias, que tienen como objetivo poder extraer información útil de grandes cantidades de trayectorias. A diferencia de simples objetos geométricos como puntos o segmentos de recta, las trayectorias son objetos más complejos, para los cuales muchas técnicas algorítmicas han sido desarrolladas durante la última década. Comentaremos ejemplos concretos de problemas de análisis de trayectorias, como el cálculo de una trayectoria que represente a un conjunto de objetos que se mueven de manera similar, o la generación automática de mapas a partir de trayectorias GPS.

Financiado por Gen. Cat. 2017SGR1640, y MINECO a través del proyecto MTM2015-63791-R y del programa Ramón y Cajal.

Why $\{6, 3, 3, -5, -5\}$ is the spectrum of a weighted digraph but not of a weighted graph?

Miriam Pisonero

Universidad de Valladolid / IMUVA

mpisoner@maf.uva.es

Abstract. The **RNIEP** (Real Nonnegative Inverse Eigenvalue Problem) is the problem of characterizing all possible real spectra of weighted digraphs. If in the RNIEP we require the digraph to be a graph, we have the **SNIEP** (Symmetric Nonnegative Inverse Eigenvalue Problem). For a long time it was thought that both problems were equivalent, but in 1996 Johnson-Laffey-Loewy set out that both problems are different and in 2004 Egleston-Lenker-Narayan proved that they are different for spectra of size greater than or equal to 5. A complete solution of both problems is known only for spectra of size $n \leq 4$, and for $n = 5$ when there are no loops. For $n \leq 4$ the most basic necessary conditions given by the Perron-Frobenius theory are also sufficient. We overview these known results, that come from the context of nonnegative matrices, and give a new method to rule out many unresolved spectra of size 5. Since 2007 it was known that $\{6, 3, 3, -5, -5\}$ is the spectrum of a weighted digraph and our method shows that can not be the spectrum of a weighted graph.

Joint work with C. R. Johnson and C. Marijuán.
Supported by Consejería de Educación de la Junta de Castilla y León (Spain) VA128G18.

Discrete mathematics apply as time goes by

David Orden

Universidad de Alcalá

david.orden@uah.es

Abstract. Back in the 80s, experts recommended that all students in sciences and engineering should be required to take some Discrete Mathematics as undergraduates [3]. Thirty years later, this talk will show two cases of application of Discrete Mathematics to problems in science and engineering, ubiquitous in our daily lifes. On one hand, a new method for processing data from tasting sessions, SensoGraph [1], which instead of the statistical methods traditionally used exploits proximity graphs and graph drawing algorithms for easier and faster processing of larger amounts of data. On the other hand, a new graph coloring problem, spectrum coloring [2], which helps to choose channel assignments in Wi-Fi networks in order to minimize the interference and thus maximize the perceived throughput.

Referencias

[1] D. Orden, E. Fernández-Fernández, J. M. Rodríguez-Nogales, and J. Vila-Crespo, Testing SensoGraph, a geometric approach for fast sensory evaluation, *Food Quality and Preference* **72** (2019), 1—9.

[2] D. Orden, J. M. Gimenez-Guzman, I. Marsa-Maestre, and E. de la Hoz, Spectrum Graph Coloring and Applications to Wi-Fi Channel Assignment, *Symmetry* **10** (2018), 65.

[3] M. J. Siegel, *Teaching Mathematics as a Service Subject.* In A. G. Howson, J. P. Kahane, P. Lauginie, E. Turckheim (Eds.), Mathematics as a service subject, Cambridge University Press, 1988, 75–89.

Joint work with the coauthors of [1] and [2].
Supported by project CCGP2017-EXP/015 of the University of Alcalá, by project MTM2017-83750-P of the Spanish Ministry of Science (AEI/FEDER, UE), and by H2020-MSCA-RISE project 734922 - CONNECT.

Los escutoides: mi experiencia en un trabajo multidisciplinar

Alberto Márquez

Universidad de Sevilla

almar@us.es

Resumen. En julio de 2017 salió publicado nuestro artículo [1]. Dicho trabajo ha sido el fruto de una colaboración, insólita para mi, con biólogos, informáticos y físicos. Quisera comentar algunos aspectos de dicha colaboración, los intentos fallidos, las aproximaciones más válidas y discutir sobre las posibilidades que ello conlleva, las dificultades encontradas, así como los problemas que nos pueden proporcionar a las matemáticas.

Referencias

[1] Gómez–Gálvez et al. *Scutoids are a geometrical solution to three-dimensional packing of epithelia.* Nature Communications 9(1), 2960, 2018

Parcialmente financiado por el proyecto BFU2016-74975-P. MEyC

On Stanley's tree isomorphism problem and the rooted U-polynomial

Anna de Mier

Universitat Politècnica de Catalunya

anna.de.mier@upc.edu

Abstract. The U-polynomial (Noble and Welsh, 99) and the chromatic symmetric function CSF (Stanley, 95) are strong graph invariants that generalize other well-known invariants, like the chromatic, matching and Tutte polynomials. When restricted to trees, the U-polynomial and the CSF are equivalent. Stanley asked in 1995 whether the CSF determines trees up to isomorphism, or, equivalently, if there exist non-isomorphic trees with the same CSF. So far it is known that all trees up to 29 vertices are distinguished by their CSF (Heil and Ji, 18), and that the same is true for caterpillars and other smaller classes (Aliste-Prieto and Zamora, 14; Loebl and Sereni, 18; Martin, Morin and Wagner, 08).

We present some contributions towards solving Stanley's question. In particular, for any integer $k \geq 2$ we give pairs of trees whose U-polynomials agree in all terms up to degree k. For this, we introduce a rooted version of the U-polynomial, which enables us to do some recursive computations not possible with the unrooted version. We will also discuss the related question of whether there exist pairs of trees whose U-polynomials agree when one sets all variables to zero, except for the first k.

Joint work with José Aliste-Prieto and José Zamora

Financiado por el proyecto MTM2017-82166-P.

A superlinear lower bound on the number of 5-holes

Irene Parada

Graz University of Technology, Austria

iparada@ist.tugraz.at

Abstract. Let P be a finite set of points in the plane in *general position*, that is, no three points of P are on a common line. We say that five points from P form a 5-*hole in* P if they are the vertices of a convex pentagon containing no other points of P. For a positive integer n, let $h_5(n)$ be the minimum number of 5-holes among all sets of n points in the plane in general position.

Despite many efforts in the last 30 years, the best known asymptotic lower and upper bounds for $h_5(n)$ have been of order $\Omega(n)$ and $O(n^2)$, respectively. We present that $h_5(n) = \Omega(n \log^{4/5} n)$, showing the first superlinear lower bound on $h_5(n)$.

A crucial step in the proof is the following structural result. If P is partitioned by a line ℓ into two subsets of size at least 5, then either ℓ intersects the convex hull of some 5-hole in P or one side is in convex position. This proof is computer-assisted.

Referencias

[1] O. Aichholzer, M. Balko, T. Hackl, J. Kynčl, I. Parada, M. Scheucher, P. Valtr, and B. Vogtenhuber. A superlinear lower bound on the number of 5-holes. In *Proceedings of 33rd International Symposium on Computational Geometry (SoCG)*, vol. 77, pp. 8:1–8:16, 2017.